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A Test of the Use of a Program of Instruction in Basic Mathematics Requiring Only Minimal Reading Skills for Use as a Remedial Tool for College Freshmen. Final Report.

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College freshmen who have poor educational backgrounds are found to be deficient in mathematics and often also lack skills in English. Their difficulty with basic mathematics is further aggravated by traditional teaching techniques that demand extensive reading ability and a thorough knowledge of English. At Morgan State College, a non-verbal program of instruction for remedial mathematics was developed which requires limited reading skills and allows students to progress at their own pace. A 2-phase study was conducted to assess the program's applicability in teaching. Selected participants were freshmen who scored below the twenty-fifth percentile on the qualitative and quantitative sections of their college entrance examinations. In the first phase the students were divided into an experimental group that utilized the program and a control group that was taught by the lecture-and-text method. The second phase was similar to the first, but weak elements of the program had been improved. The 2 groups in both phases worked 3 hours per week for 10 weeks. Programed instruction proved to be as effective as traditional teaching methods in the first phase, and superior to them in the second. It was also observed that the experimental groups progressed faster than the control groups. It is felt that this program of instruction has the potential for providing effective remedial mathematics instruction at the college level. (WM)

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Final Report

# A Test of the Use of a Program of Instruction in Basic Mathematics Requiring only Minimal Reading Skills for Use as a Remedial Tool for College Freshmen.

Submitted to: U. S. Office of Education
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Submitted by: Institute for Research in Behavioral Technology
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## 1. ABSTRACT

The freshmen test profiles of a large segment of entering Morgan State College students indicate severe deficiencies in mathematics skills. This finding is generic to state institutions of higher education. To ameliorate the problem at Morgan State College, provision for remedial work is obtained through the Science-Education Department. However, since students entering with mathematics deficiencies frequently have, as a concomitant, English language deficiencies, remediation efforts in mathematics are hampered by the dependence of mathematics teaching on advanced English language skills.

The present study examines a possible solution to the problem through utilization of a program of instruction in basic mathematics which is essentially non-verbal and requires only minimum reading skills to achieve mastery of mathematics manipulation in arithmetic, algebra, geometry, trigonometry, measurement and graphic analysis.

## 2. STATEMENT OF THE PROBLEM

Future success in college is frequently predicated upon the mastery of English language and mathematics skills. However, students entering college with a deficiency in one, frequently demonstrate concomitantly, a deficiency in the other. This creates a dilemma in so far as the learning of mathematics too frequently depends on

an advanced capability in reading skills. The student trying to learn mathematics finds himself falling further and further behind as his learning of mathematics becomes dependent on his reading ability.

The present study examined the capability of a mathematics program developed precisely to minimize the importance of reading in the teaching of mathematics.

## 3. BACKGROUND

The importance of higher education as a prerequisite for the individual meeting his responsibility in society is apparent. The concerns of public education are no longer limited to public elementary and high schools. Indeed, each state is either in the process of, or has already instituted a complex of junior colleges and colleges, to augment the capability of its state university.

With recognition of the importance of a college education, has come an awareness of a new responsibility in college teaching. If the view is postulated that the doors of higher education must be accessible to all, then the concept of the higher education as education of the select is no longer applicable.

Colleges and universities have been concerned with education of these people who are able to demonstrate the ability of "scholarliness". Stringent testing procedures

have been developed, which, while they may not be infallible, have allowed for the selection of students on the basis of high predictability for success. The colleges, therefore, are observed as a place for the student to grow as a scholar, albeit under direction.

Conceivably this tradition of scholarly growth has a place in American education. Ideally, in the best of all possible worlds, where elementary and high schools have provided students the requisite skills, colleges and universities could all function to this end. However, we are becoming progressively aware that primary and secondary schools are not fully capable of meeting the needs of all of their population. The children of the poor, the children of our minority groups, living within the constrained environments of economic and social deprivation, do not typically receive the education necessary to enter the independent world of select college experience. That this group contains a potential which must be developed is aptly demonstrated by members of the group who manage to overcome the barriers and demonstrate proficiency.

To date, the state colleges and universities, while not strictly adhering to the selective procedures of our major private schools, have not produced a new view of the function of teaching responsibility in higher education. Though their selection procedures may

not be as stringent as that of the private universities, the fact that they have not produced avenues of remediation is evidenced by the high intital drop-out rate found at most state institutions.

While committing themselves to free or inexpensive public education, they have maintained a philosophy which provides little, if any recourse to the student who has not as yet learned to be a student. Today, more than ever before, we are aware of the necessity for developing curricula and techniques to provide for a necessary solution to these problems.

Morgan State College, because of its unique position, has long been concerned with the development of programs designed to ameliorate the plight of students emerging from culturally deprived backgrounds. The college has consistently produced successful students from among the population whose success probability upon entrance has been exceedingly low. Students scoring below the 25th percentile on standardized tests have been afforded an opportunity to develop their natural potential and have left Morgan able to compete with graduates from major colleges and universities.

The college has maintained its tradition by continuously and conscientiously concerning itself with the examination and utilization of new techniques for use with its population. It has experimented with, and presently uses, a three track freshman

curriculum. Various projects have been carried out in an attempt to develop processes and procedures that will improve students' capability for independent study. Some faculty members in cooperation with the college's Programed Learning Project, have attempted, with some success, the reorganization of their classes according to the principles of Behavioral Technology.

As with most colleges, Morgan State has a freshman English program which attempts to overcome the entrance deficiencies of its student population. Remediation in mathematics is provided during an introductory science education course, the first ten weeks of which are directed to the teaching of basic mathematics. The course begins with the addition of positive and negative numbers and takes the student through basic trigonometric functions. However, examination of the existing procedures for the teaching of mathematics at this level suggests a particular dilemma. Present mathematics textbooks typically require extensive reading skills. Since these same students enter with reading difficulties, to expect them to learn mathematics through a method necessitating reading capabilities beyond their present level, makes the teaching of remedial mathematics somewhat paradoxical. What one finds is a student becoming more and more confused and less and less effective as his mathematics mastery becomes more dependent upon reading skills. This is further compounded by the

goals of the science-education program which requires not only mastery of the mathematics skills, but mastery of their particular application in science areas requiring a high degree of verbal skills.

An examination by the author[1](See Appendix A) has led to the development of a possible solution to the problem by the development of a mathematics program[2] essentially non-verbal in nature. A description of the program appears as Appendix B.

The mathematics program was written and preliminary validation indicated its potential as a remedial technique for use with the intended target population. This report details the results of a series of validation studies designed to examine the potential of a Mathematics Program of Instruction requiring limited reading skills in teaching mathematics to students with poor educational backgrounds.

## 4. PROCEDURES

This study was carried out in two phases. Phase 1 consisted of a preliminary validation whose purposes were to determine the applicability of the program of instruction in teaching mathematics to low achieving college students and also to determine the weak points in the program. Subsequently, the program was re-edited and a second

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[1]Block, A.H., A Strategy Underlying the Development of A Remedial Program in Basic Mathematics for College Freshmen. Annual Report of the Doris Duke Project in Programed Instruction-1965. Morgan State College, Baltimore, Md. 1965.

[2]Block, A.H., Colligan, R.B., Aronson, J., Programed Mathematics for Physical Science, Experimental Edition. Morgan Press, Inc. Baltimore, Md. 1966.

validation undertaken.

Phase I: Phase I consisted of a preliminary validation of the effectiveness of the existing program versus traditional teaching techniques. In addition, an item analysis of program errors was undertaken to identify weak elements requiring further edition.

Method: 160 college freshmen entering Morgan State College in the Fall and Spring semesters during the 1965-66 school year were selected as subjects for this study. These subjects were selected according to evidenced difficulties in mathematics and reading on the basis of their ATC examination, i.e., subjects were selected from among entering freshmen scoring between the fifth and fourteenth percentile in the standard range of the ATC examination.

Subjects were assigned randomly to one of two groups and equivalence of each group further tested by the use of a mathematics achievements pretest.

One group was designated experimental and the other control. The experimental group utilized the program of instruction in mathematics in lieu of the textbook situation utilized by the control group. The control group used a standard mathematics textbook[3] and attended three one-hour lectures per week.

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[3]Kruglak, H., Moore, J.T., Basic Mathematics for the Physical Sciences. McGraw Hill. 1963.

The experimental group worked on the program for three hours per week in a classroom where an instructor was available to answer any questions they may have concerning the materials. In addition, the experimental group was allowed to take the programs home and utilize them in the same way the text-books were utilized by the control group.

Upon completion of the ten-week session both groups were tested with alternate forms of a specially prepared mathematics achievement test which had also been used as the pretest.

Results: Normal college attrition rate resulted in the groups being reduced somewhat. Dropouts and withdrawals resulted in a loss of ten students by the experimental group and 21 from the control group. This left an experimental population of 59 controls and 70 experimentals.

| | Pretest | | Post Test | | | |
|---|---|---|---|---|---|---|
| | Mean | S.D. | Mean | S.D. | t | p |
| Experimental | 22.69 | 11.35 | 43.44 | 14.88 | 3.247 | .05 |
| Control | 23.20 | 10.95 | 43.14 | 13.79 | 2.919 | .05 |
| | $t = 0.0112$ | | $t = 0.0369$ | | | |
| | p = ns | | p = ns | | | |

Table 1: Means and Standard Deviations for Pre- and Post-Test Data for both Experimental and Control Groups.

Table 1 presents means and standard deviations for pre- and post-test data for both experimental and control groups. T-tests computed for pretest data indicated no significant differences between groups. T-tests computed for both experimental and control groups between pre- and post-tests were significant. ($p > .05$) T-tests computed for post-test scores between experimentals and controls indicated no significant differences between groups.

Discussion: The results of Phase 1 demonstrate that students utilizing programed instruction can achieve equally as well as those utilizing traditional instruction.

While no data was taken concerning total amount of time expended utilizing the textbook, versus the program of instruction, it should be noted that by the eighth week all students utilizing the program had completed all of their work suggesting that programed instruction has some additional value in terms of allowing students to progress at their own pace.

The item analysis suggested large areas of the program needed re-editing. This was carried out and in addition a set of unit tests were added to insure that the program was performing at a minimum equally as well as traditional classroom

instruction.

Parenthetically, it should be noted that when research is carried out within the existing framework of a college curriculum, particular attention must be paid in terms of one's responsibility to students. In attempting to determine whether or not new techniques or materials are feasible, one must provide a surveillance system such that if the technique is proved ineffective, remediation may be provided in time to insure that the student undergoing experimental treatment will suffer no academic loss.

Phase 2: Subsequent to Phase 1, an item analysis was performed on student response in the programed materials to determine weaknesses of the program. Based on this analysis, the program was re-edited and a further validation carried out. In addition, a series of four unit tests were developed and administered.

Method: From a sample of 697 entering college freshmen during the school year of 1966-67, 200 students scoring between the fifth and fourteenth percentile in the composite standard range of the ATC examination were selected. 100 students were assigned at random to the experimental group while the other 100 were assigned to the control group.

The equivalence of each group was further tested by use of a mathematics

achievement pretest. This test had alternate forms and was used in combination to determine pre-post test scores. Students were assigned to a form on a random basis. Whatever form was assigned as a pretest determined the form of the post-test.

The experimental group utilized the program materials while the control group utilized the standard mathematics text. As in the preceding validation, the experimental group was allowed access to the professors and permitted to use the program in the same manner as they would in an ordinary text. The controls attended three hours of standard lecture. Due to dropouts and scheduling problems, the final size of each sample group consisted of 77 subjects.

Following were hypotheses to be tested during this phase:

$H_1$- The level of learning of mathematics used programed instruction will increase from pre-post testing.

$H_2$- The level of learning mathematics by the control group will increase under conventional instruction.

$H_3$- The level of learning of mathematics by the experimental group as determined by pre-post test differences will exceed that of the control group.

Results: The pretests given to the control and experimental groups were examined to insure equivalency. The following statistical measurements were used in examining differences:

- Pearson Product Moment Correlation ($r_{ec}$)
- A standard error of differences between correlated means ($\sigma dm$)
- The z-ratio (z)
- Confidence Level(C.L.)

Table 2 presents the results of the analysis. The data indicates a high degree of correlation (r=0.96) while the low z-ratio of .81 indicates the lack of significance between the differences.

In addition to the pre and post-tests, four unit tests were given during the course of the program. Figure 1 is a graphic plot of these results. It is interesting to note that in all cases subsequent to the pretest experimental groups performed better than do the control groups.

Table 3 and Table 4 present the results of examining the pre-post differences for both the control and experimental groups. In both cases, the difference between the pre and post-test is significant.

# TABLE 2

## COMPARISONS OF EXPERIMENTAL AND CONTROL GROUPS EQUATED ON THE MATHEMATICS PRE TEST

| Measure | Experimental | Control |
|---|---|---|
| N | 77 | 77 |
| M | 23.47 | 24.32 |
| $\sigma$ | 9.92 | 10.19 |
| $\sigma_M$ | 1.14 | 1.17 |
| rec | 0.96 | |
| $D_M$ | 0.85 | |
| $\sigma d_M$ | 1.05 | |
| Z | 0.81 | |
| C.L. | INSIGNIFICANT | |

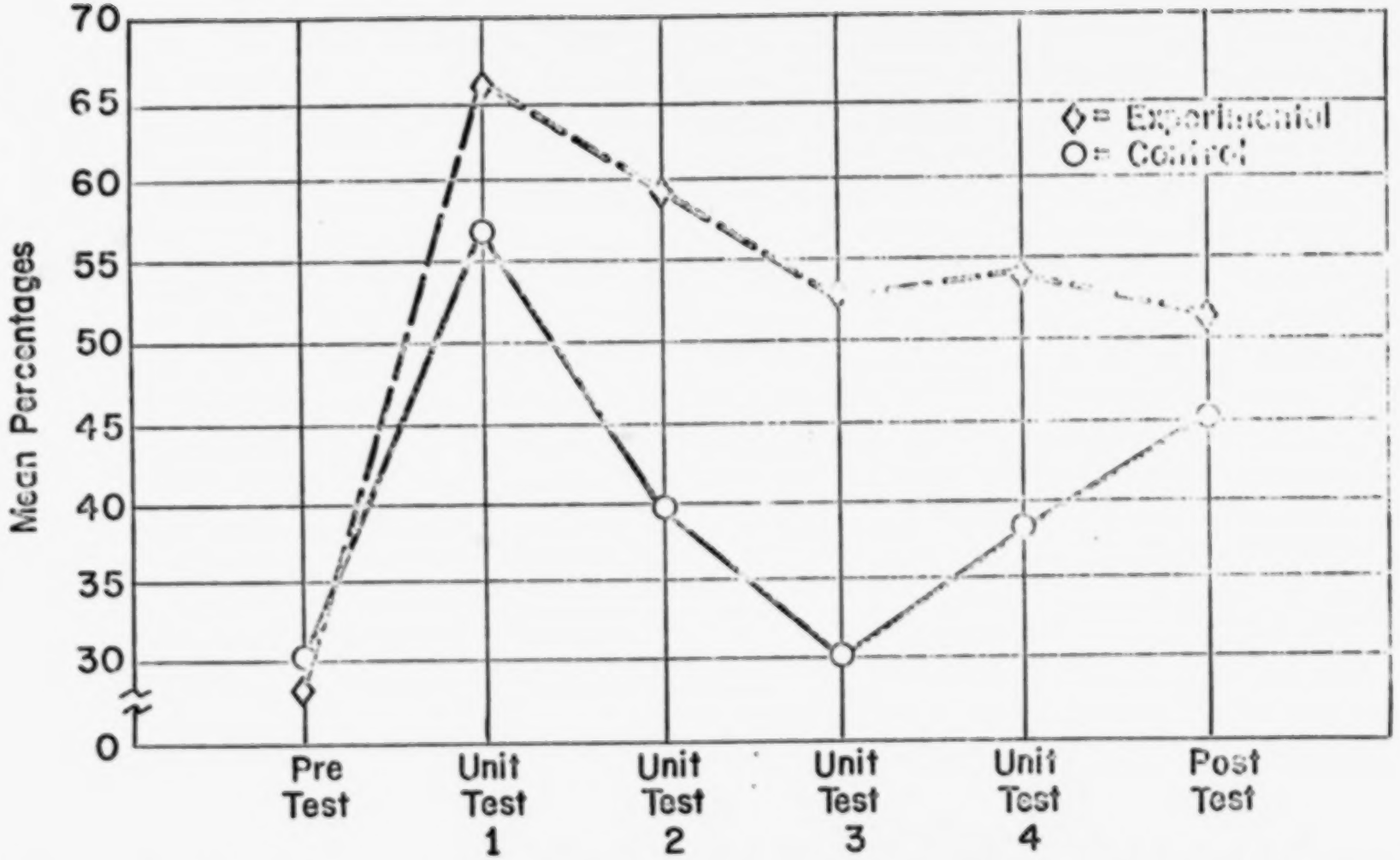


**Figure 1.** Performances of experimental and control groups on all mathematics tests based on mean percentages of the seventy-seven matched pairs.

TABLE 3

COMPARISON OF MATHEMATICS PRE TEST AND POST TEST FOR THE CONTROL GROUP

| Measure | Pre Test | Post Test |
|---|---|---|
| $N_c$ | 77 | 77 |
| $M_c$ | 24.32 | 36.51 |
| $\sigma_c$ | 10.19 | 11.61 |
| $\sigma_{M_c}$ | 1.17 | 1.33 |
| $r_{(pre,\ post)_c}$ | 0.56 | |
| $D_M(post-pre)_c$ | 12.19 | |
| $\sigma d_{M_c}$ | 1.13 | |
| Z | 10.33 | |
| C.L. | $< 0.01$ | |

TABLE 4

COMPARISON OF MATHEMATICS PRE TEST AND POST TEST FOR THE EXPERIMENTAL GROUP

| Measure | Pre Test | Post Test |
|---|---|---|
| $N_e$ | 77 | 77 |
| $M_e$ | 23.47 | 40.79 |
| $\sigma_e$ | 9.92 | 11.23 |
| $\sigma_{M_e}$ | 1.14 | 1.29 |
| $r_{(pre,\ post)_e}$ | 0.11 | |
| $D_{M(post-pre)_e}$ | 17.32 | |
| $\sigma d_{M_e}$ | 1.62 | |
| Z | 10.69 | |
| C.L. | $< 0.01$ | |

Table 5 presents the results of an examination of the post-test differences between the experimental and control groups. It should be noted that after ten weeks of instruction, the experimental group exceeds the control group in mean performance by 4.28 score points. The z-ratio for this mean difference indicates that the null hypothesis can be rejected beyond the .01 confidence level demonstrating that the level of learning under programed instruction exceeds the level of learning under conventional mathematics instruction by an amount which is both practical and statistically significant.

Discussion: An attempt was made to determine whether or not programed instruction could work effectively in meeting the needs of low mathematics achievers also demonstrating reading difficulties. This hypothesis was tested by comparing an experimental group using programed instruction with that of a control group subjected to conventional procedures (i.e., lecture and text). The results of this examination suggest the efficacy of programed instruction in that experimental group scores exceeded those of the control group.

It should be noted, however, that during Phase 1 this result was not obtained and that it resulted only after corrections were made to the program resultant

TABLE 5

COMPARISONS OF EXPERIMENTAL AND CONTROL GROUPS ON THE MATHEMATICS POST TEST

| Measure | Experimental | Control |
|---|---|---|
| N | 77 | 77 |
| M | 40.79 | 36.51 |
| $\sigma$ | 11.23 | 11.61 |
| $\sigma_M$ | 1.29 | 1.33 |
| rec | 0.33 | |
| $D_M$ | 4.28 | |
| $\sigma d_M$ | 1.52 | |
| Z | 2.82 | |
| C.L. | < 0.01 | |

from an analysis of the programs weaknesses. It is conceivable than that the differences between the experimental and control group could be maximized by further corrections to the program of instruction. In any event, the problem at hand, namely "can programed instruction be of value" appears to be demonstrated.

## 5. SUMMARY AND CONCLUSIONS

Students entering college with a deficiency in mathematics frequently demonstrate a concomitant deficiency in English language skills. This is particularly true of these students coming from backgrounds of socio-economic deprivation.

A major problem in the teaching of mathematics results from traditional methods' and textbooks' reliance on advanced English language skills. What frequently happens to youth trying to learn mathematics, who enter college with a limited background resultant from educational deprivation, is that they fall further and further behind as teaching methods and materials demand greater and greater knowledge of English. To overcome this problem a mathematics program of instruction was developed which utilized minimal English language skills. This study presents the results of an attempt to validate the program of instruction.

The study was carried out in two phases. In Phase I students utilizing a program of

instruction and having access to an instructor for questions sessions were contrasted with students subjected to conventional methods (i.e., lecture and textbook). The results showed programed instruction to be as effective as traditional methods. There was, however, a suggestion that programed instruction which permits students to work at their own pace was somewhat faster. In addition, item analysis pointed out some weaknesses in the program.

Prior to Phase 2, the program was re-edited and once again the program of instruction was contrasted with traditional methods. During this validation the program demonstrated clear superiority over the traditional procedures indicating that program of instruction which required limited English language skills may prove a solution to the problem of providing remedial mathematics instruction at the college level.

# APPENDIX A

## A Strategy Underliing the Development of a Remedial Program in Basic Mathematics for College Freshmen[1]

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In general, state financed institutions of higher education are committed to the concept of providing a college education for all. While their commitment has resulted in a more or less open door policy with respect to admissions, the high drop out rate evident in public institutions suggests that they have not evolved methods which allow the concept to obtain reality.

Two contravening notions are most probably responsible for the existing situation. On one hand, there is the awareness that the opportunity for a college education should be afforded to everyone, while on the other hand, there remains a historical tradition of colleges as a shaping grounds for the best equipped. This dilemma is further confounded by our awareness that elementary and secondary schools are not preparing students able to meet the requirements of college level work. It is becoming more and more evident that a minority segment of our population, that is the children from socio-economically deprived environments, do in fact receive short shrift where education is concerned.

Morgan State College has long been aware of, and concerned with, just these problems. As with most state colleges, entrance requirements at Morgan State College are limited. Acceptance is based on the award of a high school diploma and a graduating class standing anywhere in the upper sixty-six percent. Accordingly, we do

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[1] This paper is reprinted from the Annual Report of the Doris Duke Project in Programed Instruction, 1965, and is not to be quoted or reprinted without permission of the author.

accept students whose scores on college entrance examinations are well below the twenty-fifth percentile. Many of these students drop out during or shortly after their first year in college. An immediate solution to the alleviation of the drop out rate would be the establishment of more restrictive entrance standards. However, since many students from this population have the potential to do college level work, denying them the opportunity would continue to perpetuate the constraint within which this population is maintained. Rather than developing more stringent entrance requirement tests, we must develop curricula and methods which allow for the emergence of long constrained potential.

The Doris Duke Project in Programed Instruction is an attempt to come to grips with the problem of providing remediation at the college level. Prime among the Project's efforts is the development of a program of instruction for remedial mathematics.

## POPULATION

The target population of the program is entering freshmen who score below the twenty-fifth percentile on the qualitative and quantitative sections of their college entrance examinations. These students typically demonstrate major deficiencies in arithmetic and English language skills. Our initial attempt was to develop procedures and curricula which would permit these students to master mathematics skills without requiring

English language skill performance beyond their existing levels. Accordingly, it was decided to focus on the development of a mathematics program requiring no greater than fourth grade reading skills. No other tests were used to identify the population as previous data indicates the inappropriateness of the tests for this population.

STRATEGY

A number of general techniques have been developed in programed instruction. These techniques may be defined as strategies for the solution of particular kinds of behavioral problems. Basically, they revolve themselves around a consideration of the use of rules and examples. A definitive statement of these strategies has been presented by Evans, Homme, and Glaser.[2] In general, they define rules as "Statements of some generality from which substitution instances can be obtained. The substitution instances are called examples." This strategy has led Evans et al to develop a ruleg system for programming.

Some controversy has centered itself about the sequence of rules and examples in that some programmers prefer an eg-rul treatment, that is, leading from the specific to the general, to a ruleg treatment. The applicability of either of the two approaches is clear when one is concerned with teaching verbal skills. However there is some

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[2] Evans, J.L., Homme, L.E. and Glaser, R. The Ruleg System for the Construction of Programed Verbal Learning Sequences. J. of Ed. Reas., Vol. 55, No. 9, June-July, 1962.

question as to the necessity of teaching such skills where mathematics is concerned.

Mathematics has been defined as a logical language of a general nature consisting of rules and orderly deductions called laws and proofs. "Knowledge" of mathematics is said to be obtained when a student is able to verbalize those rules that make up the subsets of mathematics. Measurement of a student's knowledge is based then, on his ability to respond to the question, "What is the communicative law?" by the answer "a+b=b+a". For those who accept this as a reasonable demonstration of mastery of mathematics, the ruleg or eg-rul techniques are perfectly applicable. However, it appears to us that manipulation of the operation "a+b=b+a" is a more cogent definition of a mastery of mathematics than being able to state the rule.

Furthermore, if one accepts the afore-mentioned position, the confounding in the teaching of mathematics based upon the interdependence of English language skills may be eliminated. Within this framework a student demonstrates his knowledge of the communicative law by solving problems requiring the application of the law. This approach has underlied the development of our remedial program in mathematics.

The program was based on a preliminary analysis of student's deficiencies. Our initial goal was identification of these deficiencies and the development of techniques to ameliorate the deficiencies. So, for example, we identified simple multiplication

deficiencies as follows:

1. Lack of understanding that multiplication is the union (or addition) of like sets.
2. No knowledge of multiplication tables.

   (Knowledge of the "facts" of the multiplication table was deemed critical for efficient multiplication manipulation).

Solution of the deficiencies was accomplished in the following manner: (Refer to Book 1, Section 2, "A Program of Instruction in Basic Mathematics for Science Education").

1. Introduce simple multiplication as the union of sets without using terminology (Section 2, frames 1-6.)
2. Develop a sub routine for those students who are unable to demonstrate retention of the multiplication table. (Section 2, frame 7, measures retention. If the student is correct he by-passes the sub routine and moves to frame 18. If he is incorrect, the student enters a looped sub routine which can be repeated until mastery is demonstrated.)

In the case of multiple digits, the following transpired:

Deficiencies:

1. A portion of the target population could not multiply "n" digits

times "n" digits.

2. No knowledge of place value.

3. No knowledge of grouping.

Solutions:

1. Determine which students knew traditional techniques. (Section 2, frame 23 is the choice point frame. If successful, the student is directed to move ahead to frame 76. We decided that it would be too confusing to teach new methods to those students who had mastered traditional techniques.)

2. Develop routines which provide students insight into what the actual process of multiplication is.

    a. Teach place value beginning with units and tens.

    b. Teach the concept of re-grouping.

Examination of frames 23-76 demonstrates the way in which this was done. It should be noted that we lead the student through certain steps and then ask him to demonstrate his mastery of that material in the general sense. In all cases we do this through manipulation rather than any verbal statements. We do not for example, intend the student to be able to tell us that he has used the concept of regrouping to solve multiplication problems, but we do expect him to solve them. This approach

has been used throughout the program.

RESULTS

The program has been evaluated for preliminary revisions on three people to date. In all cases a significant difference was noted between pre and post testing. The response of one student in particular is of value to note. This student has previously failed Science-Education 113. Her score on the Science Education Mathematics Screening Test was nine out of a possible eighty-five. Her post test score was sixty-seven. In addition, she has continuously indicated her preference for this program. "For the first time I understand what it is they were trying to teach me", and "I liked the way it is written. I understand what they are saying."

# APPENDIX B

## A Description of Programed Mathematics for Physical Science

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A Description of Programed Mathematics for Physical Science.

The program has been designed to provide remediation to students demonstrating inadequate experience in mathematics and reading skills. The program covers six topics: arithmetic, algebra, geometry, trigonometry, measurement and graphic analysis.

Sequence

Following is a specification of the sequence of the program:

4.e.1. Arithmetic

4.e.1.a. Natural Numbers, Negative Numbers, Basic Arithmetic.

4.e.1. a.1. Addition of decimal numbers using signs.
a.2. Multiplication of two or more signed numbers with decimals.
a.3. Subtraction, using decimals and parenthesis.
a.4. Addition of minus numbers using like and unlike signs.
a.5. Subtraction of negative numbers.
a.6. Division of decimals, both positive and negative values.

Unit Test 1

4.e.1.b. Operations, Symbols, Decimal/Fraction relationship.

4.e.1. b.1. Order of Operations.
b.2. Changing decimals to fractions; fractions to decimals.
b.3. Factoring of numbers.

Unit Test 2

4.e.1.c. Fractions and Decimal Fractions

4.e.1. e.1. Introduction
e.2. Multiplications
e.3. Division
e.4. Multiplication and division by one
e.5. Mixed numbers
e.6. Reduction
e.7. Addition
e.8. Subtraction
e.9. Use of signs

Unit Test 3

4.e.1.d. Ratio, Proportion and Percent.

4.e.1. d.1. Ratio and proportion, inverse and direct
d.2. Percentages

Unit Test 4

4.e.1.e. Extraction of Square Roots

4.e.1. e.1. Extraction of square roots
e.2. Reciprocals

Unit Test 5

4.e.2. Algebra

4.e.2.a. Algebraic Expressions and operations.

4.e.2. a.1. The language of Algebra
a.2. Monomials: Division, addition and subtraction

Unit Test 6

4.e.2.b. Simple Products, Fractions and Factors.

4.e.2. b.1. Monomials: Multiplication
b.2. Monomials: Division, using signed exponents

Unit Test 7

b.3. Monomials: Order of operation
b.4. Monomials: Multiplication and division of fractions
b.5. Monomials: Addition and subtraction of fractions
b.6. Monomials: Operations of signed fractions.

Unit Test 8

b.7. Polynomials: Addition and subtraction
b.8. Polynomials: Multiplication
b.9. Polynomials: Factoring a common monomial factor and division by a monomial
b.10. Factoring of trinomials
b.11. Polynomial Fractions: Simplification, addition, subtraction, multiplication and division

Unit Test 9

4.e.2.c. Linear and Quadratic Equations

4.e.2. c.1. Linear equations with one unknown
c.2. Quadratic Equations

Unit Test 10

4.e.2.d. Functions and the Graphs of Functions
4.e.2.e. Simple Power Functions and Variations

Unit Test 11

4.e.2.f. The Linear Function
4.e.2.g. Simultaneous Linear Equations

Unit Test 12

4.e.2.h. Exponents and Radicals

Unit Test 13

4.e.2.i. The Binomial Theorem

Unit Test 14

4.e.2.j. Logarithms and Logarithmic Functions

Unit Test 15

4.e.3. Geometry

4.e.3.a. Basic Concepts:Planes

4.e.3. a.1. Angles
a.2. Triangles

Unit Test 16

a.3. Parallel and perpendicular lines
a.4. Parallelograms

Unit Test 17

a.5. Circles and arcs
a.6. Perimeters and areas of plane figures

Unit Test 18

4.e.4. Trigonometry

4.e.4.a. Basic Trigonometric Functions

Unit Test 20

4.e.4.b. Solutions of Right Triangles
4.e.4.c. Solutions of Oblique Triangles

Unit Test 21

4.e.4.d. Radian Measure
4.e.4.e. Extrapolation and Interpolation

Unit Test 22

4.e.5. Measurement and Graphic Analysis

4.e.5.a. Experimental Error
4.e.5.b. Accuracy
4.e.5.c. Precision
4.e.5.d. Simple Error Analysis
4.e.5.e. Elements of a Good Graph
4.e.5.f. Empirical Equations